

CR 151213

MCDONNELL DOUGLAS TECHNICAL SERVICES CO.  
HOUSTON ASTRONAUTICS DIVISION

SPACE SHUTTLE ENGINEERING AND OPERATIONS SUPPORT

DESIGN NOTE NO. D0410-007

A DETAILED DESCRIPTION OF THE SEQUENTIAL PROBABILITY  
RATIO TEST FOR 2-IMU FDI

MISSION PLANNING, MISSION ANALYSIS AND SOFTWARE FORMULATION

24 MARCH 1976

This Design Note is Submitted to NASA Under Task Order  
No. D0410, Task Assignment A, Contract NAS 9-13970.

PREPARED BY:

T. M. Rich  
T. M. Rich  
Engineer  
488-5660, Ext. 293

APPROVED BY:

W. J. Mason  
W. J. Mason  
Task Manager  
488-5660, Ext. 293

APPROVED BY:

Peggy M. Dunge  
P. M. Dunge  
MPB Work Package Manager  
488-5660, Ext. 222

APPROVED BY:

W. E. Hayes  
W. E. Hayes  
MBS Manager  
488-5660, Ext. 266

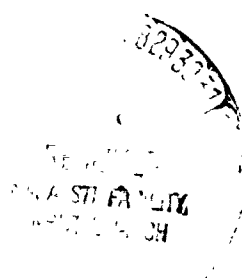
N76-33276

Unclass  
05735

63/1

CSCL 22B

(NASA-CR-151013) A DETAILED DESCRIPTION OF  
THE SEQUENTIAL PROBABILITY RATIO TEST FOR  
2-IMU FDI (McDonnell-Douglas Technical  
Services) 30 P HC \$4.00



## 1.0 SUMMARY

This design note describes the sequential probability ratio test (SPRT) for 2-IMU FDI. The SPRT is a statistical technique for detecting and isolating soft IMU failures, originally developed for the strapdown inertial reference unit (SIRU) and later adapted at Draper Labs to redundant gimbaled IMU's. This note documents the equations and describes the version of SPRT that will be used for analysis at JSC/MDTSCO. Though some of the theory is discussed, a complete coverage of SPRT theory is beyond the scope of this report and is relegated to the reference material cited. The flowchart of a subroutine incorporating the 2-IMU SPRT is included, and is referred to in the text for illustration purposes. Neither test case data nor performance evaluation is included, as these will be published separately.

## 2.0 INTRODUCTION

Last September the SPRT algorithm was baselined at the Level B OFT Entry SDR to perform the onboard 2 and 3 IMU FDI testing during shuttle entry. In order to both develop and verify the method, a SPRT subroutine was added to the IMUFDI triple - IMU simulation program on the JSC Univac 1110 computer. The version of SPRT documented in this report has evolved out of the development work to date. Future developments in SPRT theory are anticipated and will be documented in forthcoming reports. The objective of the studies to date has been to optimize performance and sensitivity of the algorithm. With this approach,

the margins between actual performance and external performance requirements may be easily established.

### 3.0 IMPLEMENTATION DESCRIPTION

This section describes the SPRT algorithm for performing the 2-IMU FDI function with skewed shuttle IMU's. The SPRT subroutine of the IMUFDI program (flowcharted in Appendix 1) is one of several versions of the SPRT algorithm that have been explored in the past two years. The version described in this report is felt to be both the most theoretically straightforward version, and also the version most akin to the data tracking test (References 1 and 2), thereby enabling an accurate comparison between the two methods. The Draper Labs version (Reference 3) differs slightly from this implementation in some areas.

The basic idea of this FDI method is to geometrically resolve a failure direction to one of four uniquely oriented accelerometers or gyros contained in a pair of skewed IMU's (Reference 4). Each IMU has one planar (XY) and one single axis (Z) accelerometer, for a total of 4 accelerometer instruments per IMU pair. In addition, each IMU has 2 planar gyros (XY, ZR) with the 4th redundant (R) axis oriented in the plane of the XY gyro,  $12^\circ$  from the X axis. It is assumed for the 2-IMU SPRT algorithm that the ZR gyro can fail only along the Z axis, since any failure with a component along the R axis would be detected by the redundant gyro monitor test (Reference 5). Gyro data from the Z axis can therefore be treated

as if it were sensed by a single axis gyro. This assumption brings the gyro set into correspondence with the accelerometer set (i.e., one XY and one Z axis instrument per IMU), thereby allowing the same algorithm to be used for both accelerometer and gyro FDI. There are minor differences in error measurement calculation and thresholding which will be described subsequently.

Figure 1 illustrates the functional flow of the 2-IMU SPRT. The method is applicable to any pair of IMU's; however, for the sake of notational clarity, IMU's #1 and #2 are singled out in this report. Major functions are discussed individually as follows:

### 3.1 Error Measurement Calculation

This function takes output data from the IMU's ( $\Delta V$ 's, gimbal angles) and forms a vector equal to the discrepancy (or error) between two IMU's. This error vector is then output in the coordinate system of each IMU stable platform.

#### 3.1.1 Gyro

The gyro error measurement is the "total relative misalignment" vector, consisting of the off-diagonal elements of the matrices Q and C as shown in Figure 2.a. The variables referred to in this figure are described as follows:

$B_{jN}(n)$  = is the Euler angle transformation matrix from IMU stable platform #j to the navigation base, computed using the set of gimbal angles read from IMU #j at the  $n^{th}$  time step.

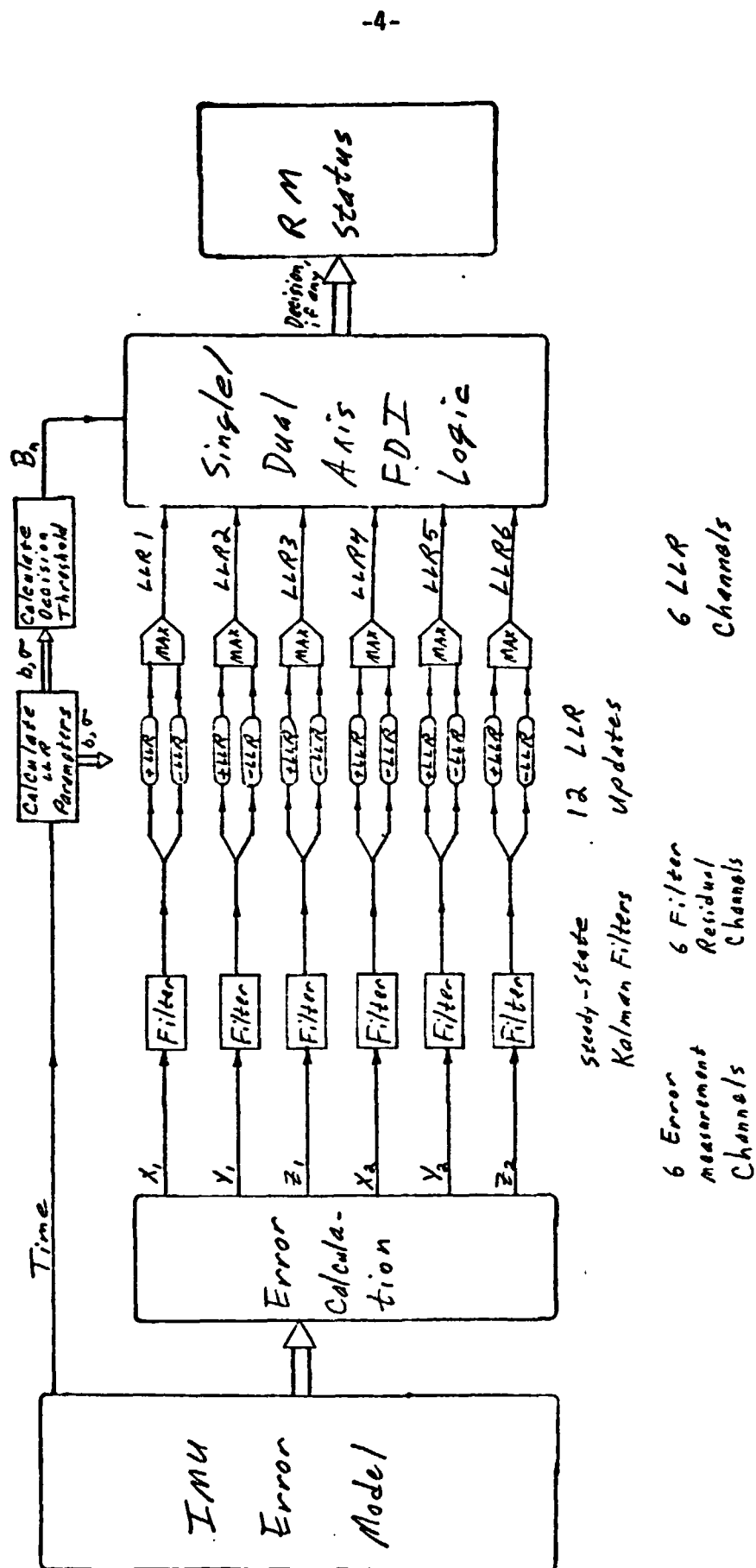


Figure 1. 2-IMU SPRT Block Flowchart

$B_{12}(n) = B_{2N}^T(n) B_{1N}(n)$  is the gimbal angle derived transformation from IMU #1 to IMU #2 stable platforms at the  $n^{th}$  time step.

$Q = B_{12}^T(1) B_{12}(n)$  is the skew symmetric matrix whose off-diagonal elements form the total relative misalignment vector  $(\theta_{u,v,w})$  in IMU #1 coordinates.

$C = B_{12}(1) B_{12}^T(n)$  is the skewsymmetric matrix whose off-diagonal elements form the total relative misalignment vector  $(\theta_{x,y,z})$  in IMU #2 coordinates.

In the real time computer the six small misalignment angles,  $\theta_{u,v,w}$  and  $\theta_{x,y,z}$ , can be computed directly as dot products of rows and columns of the transformations  $B_{12}(1)$  and  $B_{12}(n)$ .

### 3.1.2 Accelerometer

The accelerometer error measurement is the "incremental  $\Delta\Delta V$ " vector formed by differencing IMU sensed  $\Delta V$ 's in the coordinate systems of each IMU cluster, as shown in Figure 2.b. The current gimbal angle derived transformation,  $B_{12}(n)$ , is used for all transformations to minimize the effects of gyro drift on accelerometer FDI.

### 3.2 Whitening Filter

A first order recursive filter is used to transform the correlated IMU error measurements in to a sequence of independent samples. The theory for using a filter in this way is embodied in the widely known result from estimation theory that the residuals of an optimal Kalman

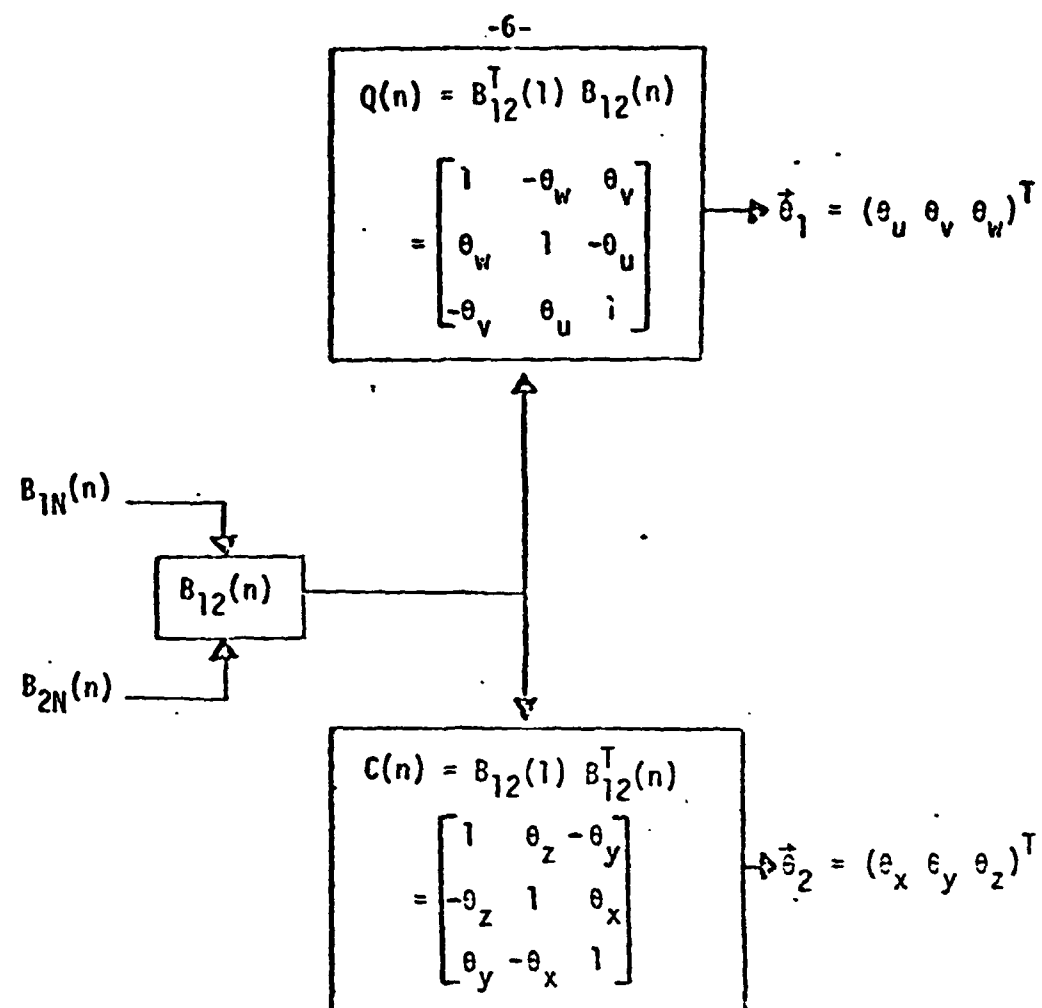


Figure 2.a) Gyro Error Measurement Calculation

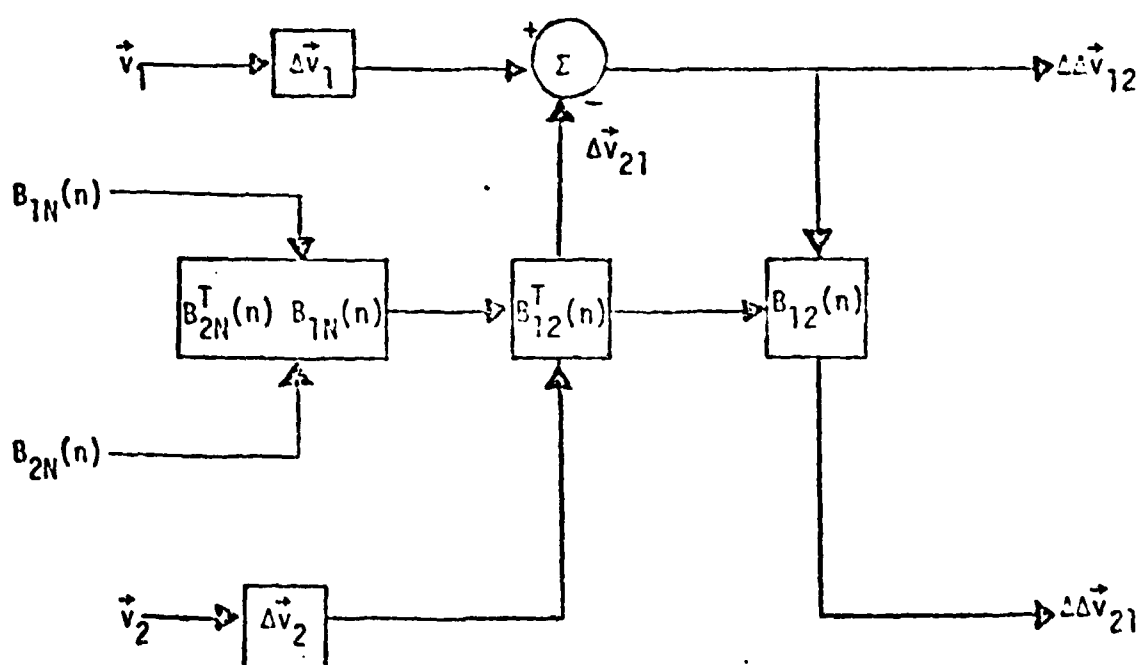


Figure 2.b) Accelerometer Error Measurement Calculation

filter constitute a white Gaussian noise sequence (References 6 and 7). In this SPRT implementation, each axis is filtered separately by a single state estimator. The outputs of the filters are the residuals formed by subtracting the filter estimate from the actual IMU error measurement. The whitening transformation performed by this filter is required because independancy of samples is assumed in formulating the LLR update equation (Section 3.4).

### 3.2.1 Filter Equations

First pass initialization:

$$\begin{aligned} S_0 &= 0 \\ \phi &= e^{(-\text{DELTAT}/\tau)} \end{aligned}$$

State propagation

$$\hat{S}_n = \phi S_{n-1}$$

Residual computation

$$r_n = Y_n - \hat{S}_n$$

State update

$$S_n = \hat{S}_n + K r_n$$

where  $S_n$  = state estimate at  $n^{\text{th}}$  time step

$r_n$  = residual at  $n^{\text{th}}$  time step (filter output)

$Y_n$  = IMU error measurement at  $n^{\text{th}}$  time step (filter input)

$K$  = filter gain (pre-mission constant)

$\tau$  = Autocorrelation time constant (pre-mission constant)

The above filter is used for both gyro and accelerometer data; the only difference being in different state update gains ( $K$ ), which are listed in section 4.2.



### 3.2.2 Pre-mission Filter Tuning

The filter gains mentioned above were computed pre-mission via a tuning procedure derived from Mehra, Reference 6, which basically states that the optimality of the filter can be measured by how uncorrelated the residuals are. In a routine written for the Univac 1110 Demand terminal, the filter gains were varied until the average autocovariance of the filter residuals over 30 Monte Carlo cycles was zeroed out. The procedure used for calculating the average autocovariance follows:

Step 1. Calculate a residual mean for each of 6 axes in each Monte Carlo Cycle between the times 305 and 1205 (this time period contains no vehicle attitude transients):

$$b_{\text{axis, cycle}} = \frac{1}{199} \sum_{\substack{t=305 \\ \text{by } 5}}^{1205} r_t$$

Step 2. Calculate the first autocovariance coefficient,  $R(1)$ , for each of 6 axes in each Monte Carlo cycle:

$$\rho_{\text{axis, cycle}} = \frac{\frac{1}{198} \sum_{\substack{t=310 \\ \text{by } 5}}^{1205} (r_t - b) (r_{t-5} - b)}{\frac{1}{199} \sum_{\substack{t=305 \\ \text{by } 5}}^{1205} (r_t - b)^2}$$

Step 3. Calculate the average autocovariance coefficient.

$$\rho_{\text{ave}} = \frac{1}{180} \sum_{\text{axis}=1}^6 \sum_{\text{cycle}=1}^{30} \rho_{\text{axis, cycle}}$$

Repeat steps 1 through 3 on residuals acquired by iterating on the filter gain  $K$  until a  $\rho_{ave}$  of approximately zero is obtained.

While computing the gyro gain of section 4.2 ( $K_{GYRO} = .54$ ) the following autocovariances were obtained:

$$\rho_{ave} = -.4\%$$

$$\rho_{min} = -34.7\% \text{ (X axis, IMU \#2, cycle \#20)}$$

$$\rho_{max} = 64.2\% \text{ (X axis, IMU \#2, cycle \#27)}$$

While computing the accelerometer gain of section 4.2 ( $K_{ACCL} = .084$ ) the following autocovariances were obtained:

$$\rho_{ave} = .1\%$$

$$\rho_{min} = -52.0\% \text{ (Z axis, IMU \#2, cycle \#23)}$$

$$\rho_{max} = 81.9\% \text{ (Z axis, IMU \#2, cycle \#30)}$$

### 3.3 LLR Parameter Calculation

The LLR update described in section 3.4 requires two parameters to compute the LLR - the classification threshold and the standard deviation for the filter residuals.

#### 3.3.1 Residual Threshold

The SPRT algorithm requires thresholds separating residuals indicative of failure operation from residuals encountered during nominal operation. As in all threshold type tests, it is desirable to have the threshold tightly fitted for greater sensitivity, yet still high enough so that no false alarms are produced under nominal operation. The failure residual thresholds used in this version of SPRT were generated by multiplying a base failure threshold (Figure 3) by a dynamic scaling factor which accounts for the filter

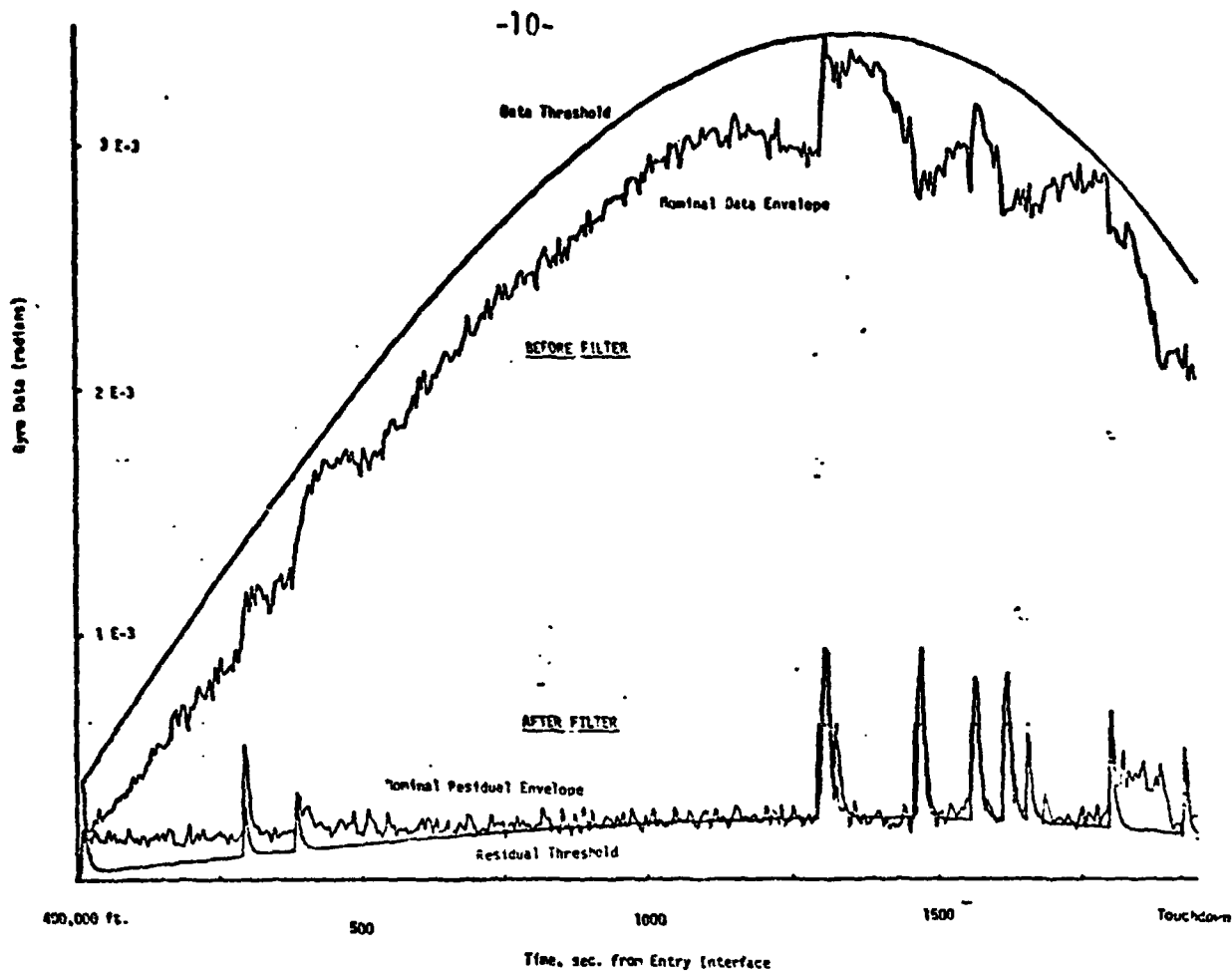


Figure 3.a) Total Relative Misalignments

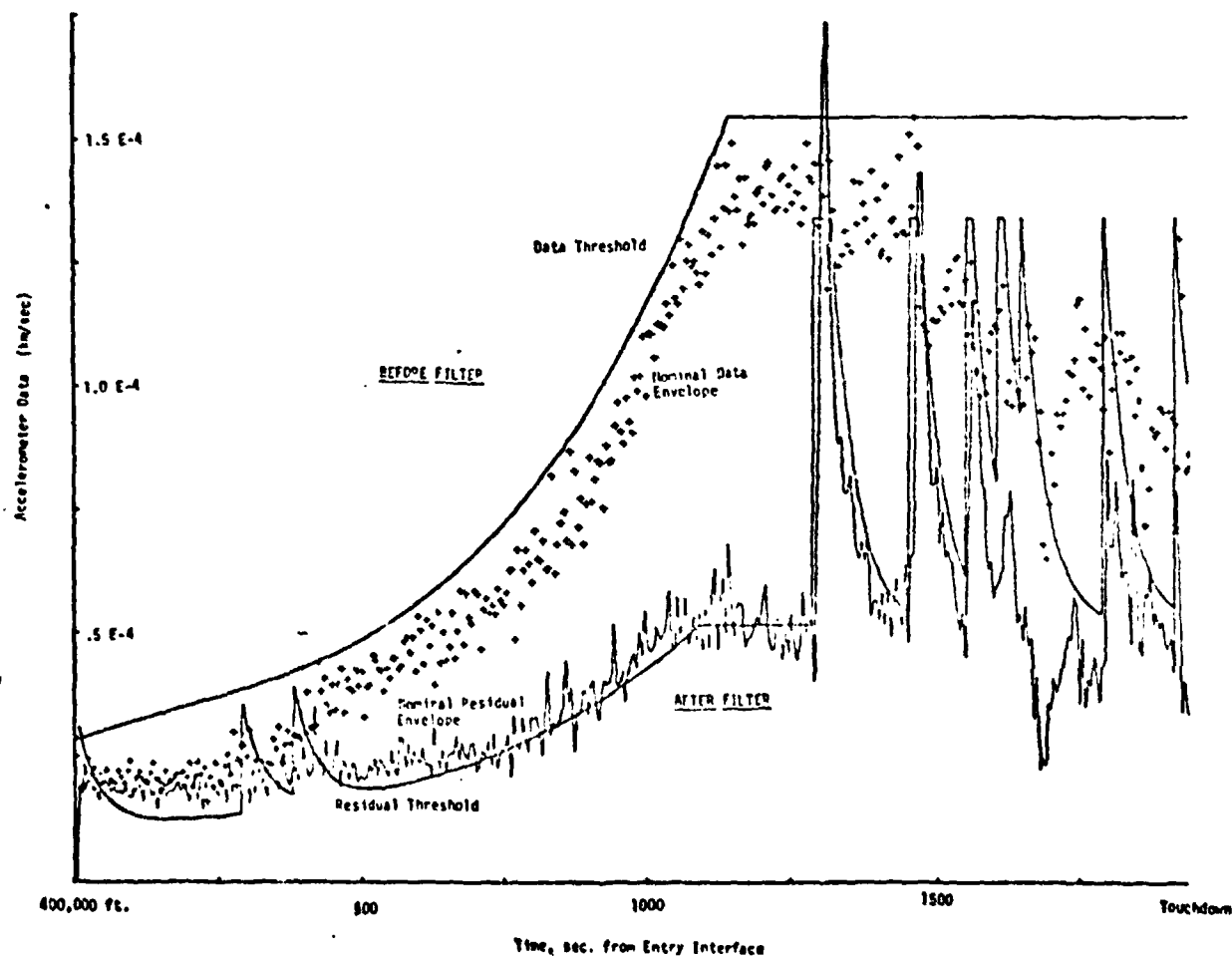


Figure 3.b) Incremental  $\Delta V$  Differences

ORIGINAL PAGE IS  
OF POOR QUALITY

effects of bias attenuation and attitude transients. The base thresholds, specified in section 4.4, are the same closely fitted thresholds used in the data tracking test (Reference 1). The scaling factor  $\Gamma$ , derived from Reference 8, is calculated by:

$$\Gamma(t) = (\Gamma_{ss} + \Gamma_n = \Gamma_{NB,n})$$

where:

$$\Gamma_{ss} = \frac{1-\phi}{1-\phi(1-K)}$$

letting  $n=t/\Delta t$ ,

$$\Gamma_n = \frac{(1-P_{NB})K\phi}{1-\phi(1-K)} (\phi(1-k))^{n-1}$$

$$\Gamma_{NB,n} = \frac{P_{NB} K\phi}{1-\phi(1-K)} (\phi(1-k))^{n-j}$$

in which  $\phi = e^{-\Delta t/\tau}$

$K$  = filter gain

$P_{NB}$  = percentage attributed to vehicle

attitude transients (Section 4.5)

$j$  is set equal to  $n$  whenever the greatest

gimbal angle change exceeds the limit

$\Delta_{Gim}$  (section 4.5)

The final threshold is then

$$BR(t) = \Gamma(t) T(t)$$

which is plotted in Figure 3.

### 3.3.2 Residual Standard Deviation

In addition to the residual threshold, the SPRT requires a residual standard deviation characterizing the "spread" of

nominal data. Standard deviations were computed at several time slots by the formula

$$\sigma(t) = \left( \frac{1}{180} \sum_{\text{axis}=1}^6 \sum_{\text{cycle}=1}^{30} \frac{1}{8} \sum_{\substack{t'=t \\ \text{by } 5}}^{t+5} r_{t'}^2 \right)^{1/2}$$

Residual standard deviations were found to be fairly constant at the values given in section 4.5.

### 3.4 Log Likelihood Ratio Updates

The likelihood ratio is the heart of the SPRT algorithm, being proportional to the probability that a failure has occurred. The log of the likelihood ratio is used to avoid computation of the EXP function at each time point. Log likelihood ratios (LLR's) are computed sequentially, modeling the filter residuals as independent Gaussian samples (not necessarily zero mean), using the residual thresholds and sigmas computed in section 3.3. In this implementation the LLR for each axis is split into two sub-LLR's, one for positive and one for negative failures, since either polarity is assumed equally likely. The LLR equations follow:

#### LLR Update

$$\Lambda_n^+ = \Lambda_{n-1}^+ + \frac{2BR(t)}{\sigma^2} (r_n - BR(t))$$

$$\Lambda_n^- = \Lambda_{n-1}^- - \frac{2BR(t)}{\sigma^2} (r_n + BR(t))$$

### LLR Reset

If  $\Lambda_n^+ < 0$  then  $\Lambda_n^+ = 0$

If  $\Lambda_n^- < 0$  then  $\Lambda_n^- = 0$

Take the biggest:

$$\Lambda_n = \max (\Lambda_n^+, \Lambda_n^-)$$

### 3.5 LLR Threshold Calculation

The LLR failure decision threshold is calculated according to Reference 9, using the residual thresholds and sigmas from section 3.3, by the following equation:

$$BR(t) = \ln (2BR^2(t)/ALPHA \sigma^2)$$

### 3.6 Single Axis/Dual Axis FDI Logic

This FDI function performs a threshold test on the 6 error measurement channels for each instrument type, both accelerometer and gyro. Each instrument (2DOF, 1DOF) is tested independently for evidence of failure. For example, if either the X or the Y gyro measurement from IMU #1 is out of tolerance, the failure has been detected in the XY<sub>1</sub> gyro. Figure 4 shows the logic by which individual instrument failure detection test are combined to isolate the failure to a particular instrument. An IMU failure has been detected when at least one of the four instruments of either type, gyro or accelerometer, is out of tolerance. In the IMUFDI implementation, at least two simultaneous threshold crossings out of the four instruments is required before an IMU failure detection is registered. A single threshold crossing would be sufficient,

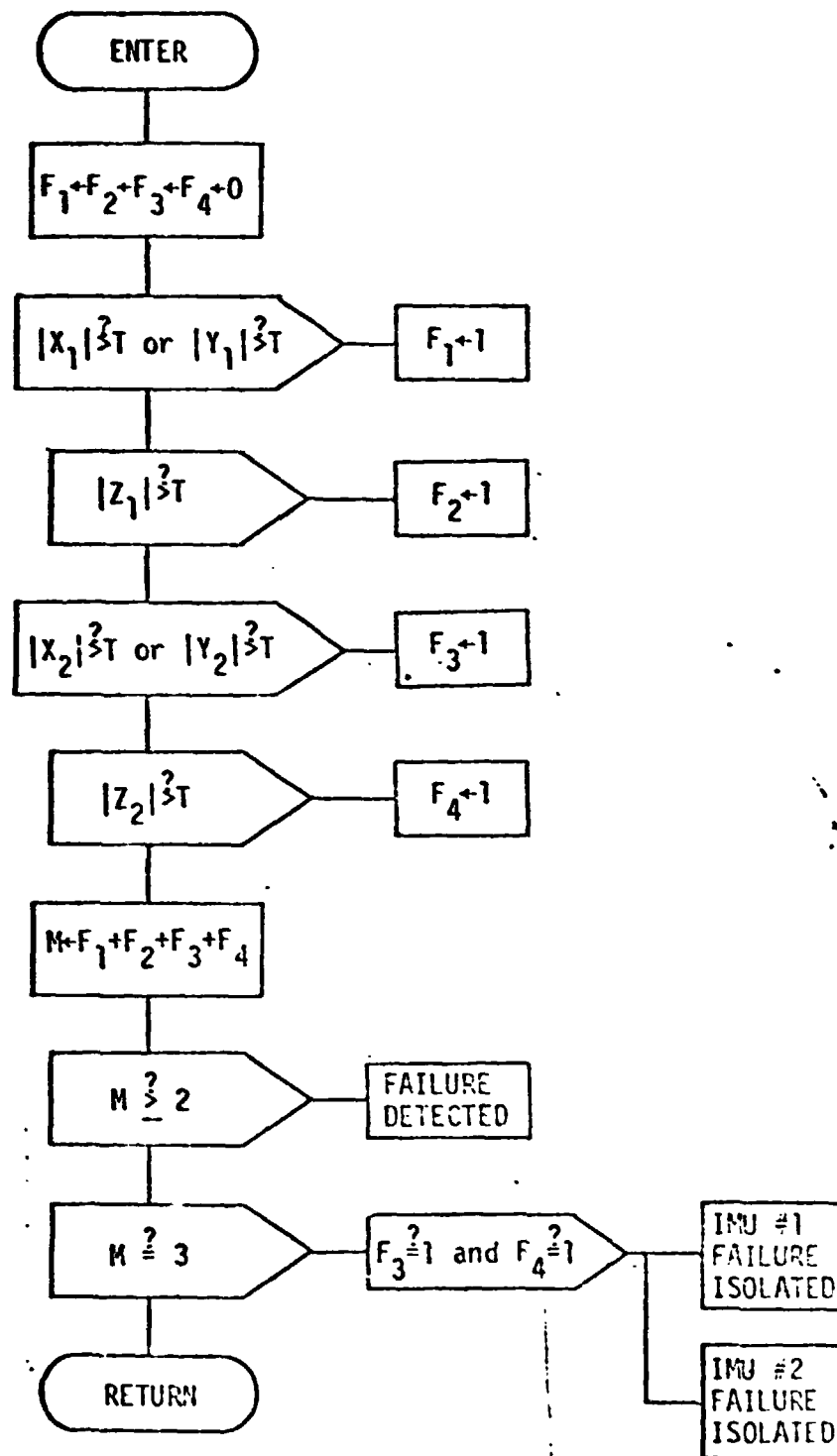


Figure 4. Single Axis/Dual Axis FDI Logic

but is also potentially more susceptible to false alarms. A failure can be isolated when exactly three of four instruments simultaneously show out-of-tolerance measurements. The IMU where the failure has been detected in both instruments is the unfailed IMU, since two detections would be indicative of a simultaneous failure - an assumed impossibility. Consequently, the IMU in which the failure has been detected on only one instrument measurement is the failed IMU. The case of four simultaneous threshold crossings (not covered in the flowchart of Figure 4) is an abnormal condition and should be flagged to the crew.

#### 4.0 RESULTS

The following paragraphs present a group of constants, pertinent to the 2-IMU SPRT algorithm, that were used in the IMEDI program to generate the results contained in Reference 10. These constants apply to the reference mission 3B entry trajectory.

##### 4.1 Time Step

$$\text{DELTAT} = 5 \text{ sec.}$$

##### 4.2 Filter Constants

Autocorrelation time constant:

$$\tau_{\text{GYRO}} = \tau_{\text{ACCL}} = 120 \text{ sec.}$$

Filter gains:

$$K_{\text{GYRO}} = .54$$

$$K_{\text{ACCL}} = .084$$



#### 4.3 Start - Stop Times

$$T_{\text{start}} = 0 \quad (400,000 \text{ ft.})$$

$$T_{\text{end}} = 1945 \text{ (Touchdown)}$$

#### 4.4 Base Failure Thresholds

Gyro:

$$TG(t) = \sum_{i=0}^3 C_i t^i \quad (\text{radians})$$

$$C_0 = 3.7000 \text{ E-4}$$

$$C_1 = 3.4967 \text{ E-6}$$

$$C_2 = -1.1786 \text{ E-10}$$

$$C_3 = -5.8263 \text{ E-13}$$

Accelerometer:

for  $0 \leq t \leq 1145$ ,

$$TA(t) = \sum_{i=0}^3 C_i t^i \quad (\text{kn/sec})$$

$$C_0 = 2.8300 \text{ E-5}$$

$$C_1 = 4.3503 \text{ E-8}$$

$$C_2 = -5.3665 \text{ E-11}$$

$$C_3 = 9.7743 \text{ E-14}$$

for  $t > 1145$ ,

$$TA(t) = TA(t=1145)$$

#### 4.5 LLR Constants

Gyro:

$$P_{NB} = .15$$

$$\sigma_G = 2.4 \text{ E-4 radians}$$

Accelerometer:

$$P_{HIB} = .8$$

$$\sigma_A = 1.2 \text{ E-5 Km/sec}$$

4.6 Mean Time Between False Alarms

$$T=5000 \text{ sec}$$

Mean False Alarm Rate:

$$ALPHA=DELTA T/T=10^{-3}$$

5.0 REFERENCES

1. T. H. Rich, "A Detailed Description of the Data Tracking Test for 2-IMU FDI", MDTSO Design Note No. D0410-005, 12 Jan. 1976.
2. T. H. Rich, "Performance Results of the Data Tracking Test for 2-IMU FDI", MDTSO Working Paper No. E914-8A-026, 27 Feb. 1976.
3. M. B. Adams, "SPRT Documentation for Level C FSSR", CSDL ISS 75-308, 27 Oct. 1975.
4. E. G. Solov and J. R. Thibodeau, "Failure Detection and Isolation Methods for Redundant Gimbaled Inertial Measurement Units", AIAA Paper No. 73-851, 20 Aug. 1975.
5. T. H. Rich, "Inertial Measurement Unit Redundant Axis BITE Sensitivity Analysis", MDTSO Design Note No. 1.4-8-001, 4 Feb. 1975.
6. R. K. Mehra, "On the Identification of Variances and Adaptive Kalman Filtering", IEEE Trans. Auto. Control, Vol. AC-15, April 1970.
7. T. Kailath, "The Innovations Approach to Detection and Estimation Theory", Proc. of the IEEE, Vol. 58, May 1970.
8. M. B. Adams, "Impact of IMU Bias Errors on IMU FDI", CSDL ISS #75 307, 16 Dec. 1975.
9. J. V. Harrison and T. T. Chien, "Failure Isolation For A Minimally Redundant Inertial Sensor System", IEEE Trans. Aerospace and Electronic Systems, Vol. AES-11, May 1975.
10. T. H. Rich, "Performance Results of the Sequential Probability Ratio Test for 2-IMU FDI", MDTSO Design Note No. D0410-008, 31 March 1976.

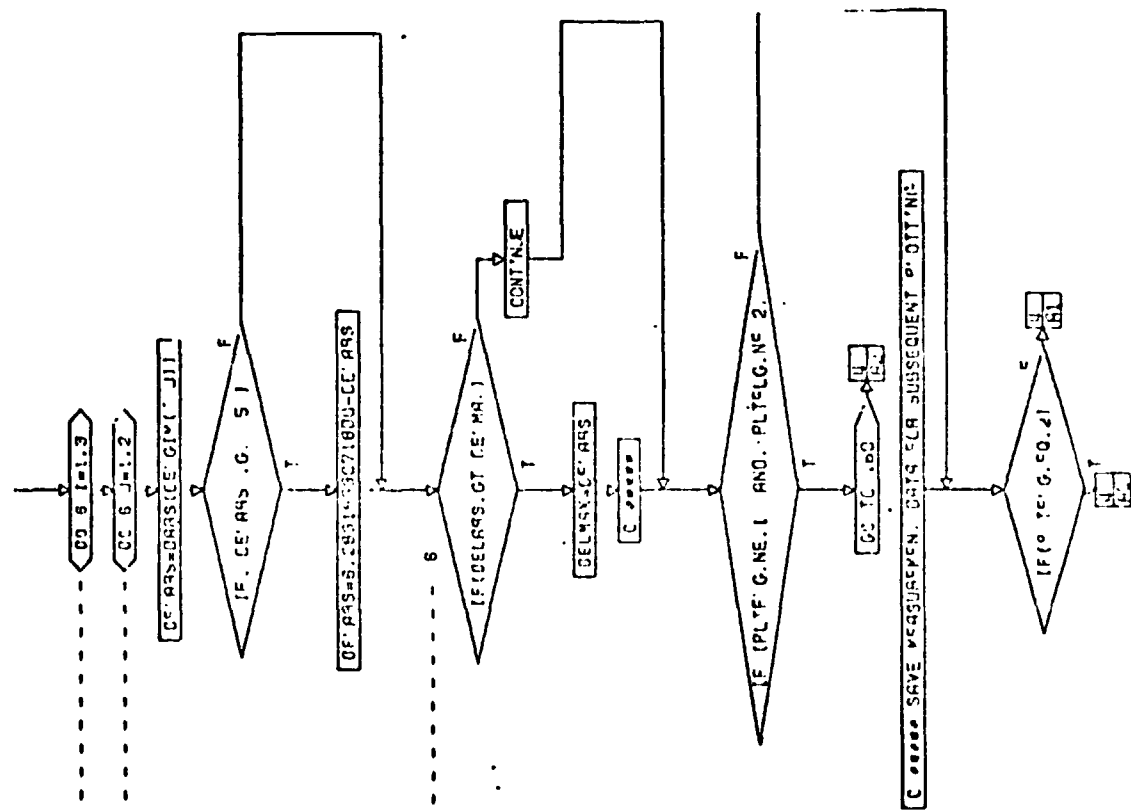
## APPENDIX 1

### Flowchart of the SPRT Subroutine

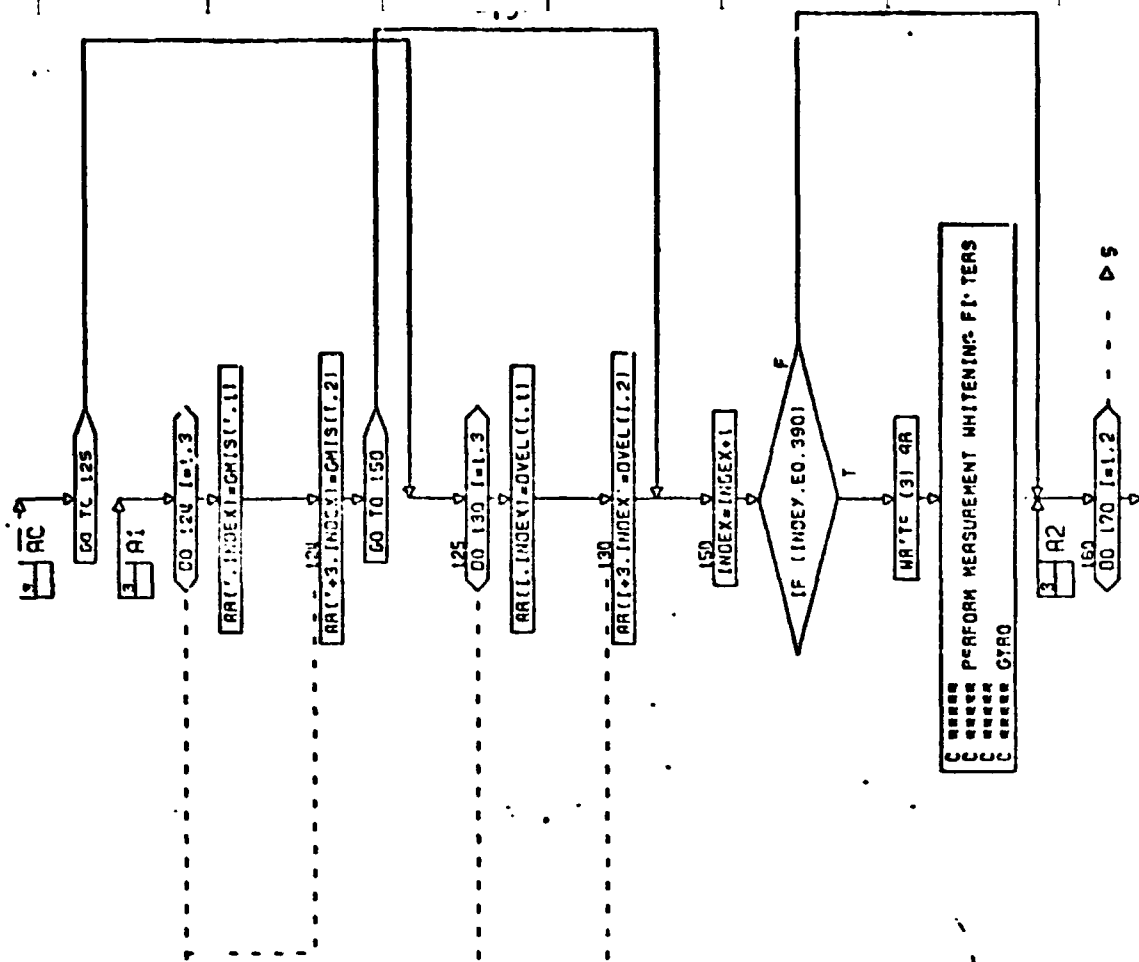
93 MO 14073

SPAT  
PG 1 OF 15

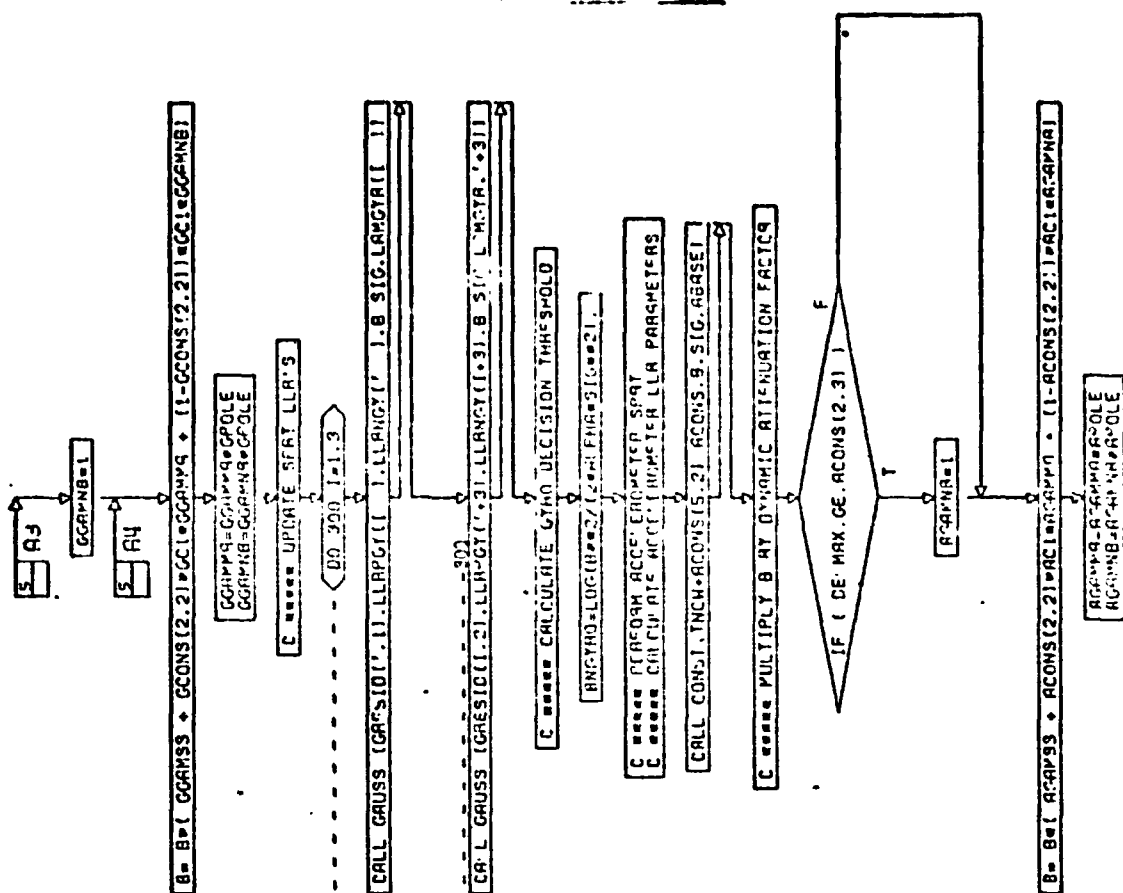
ORIGINAL PAGE IS  
OF POOR QUALITY



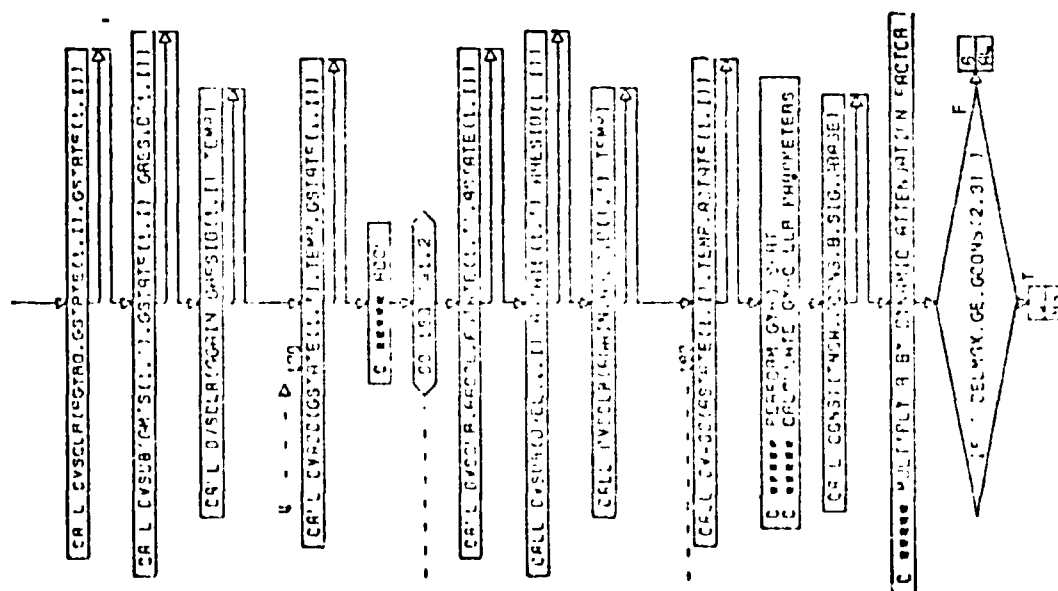
CONT. ON PG 4 SPAT  
PG 3 OF 15



CONT. ON PG 5 SPAT  
PG 4 OF 15



SPAT  
PG 8 OF 15

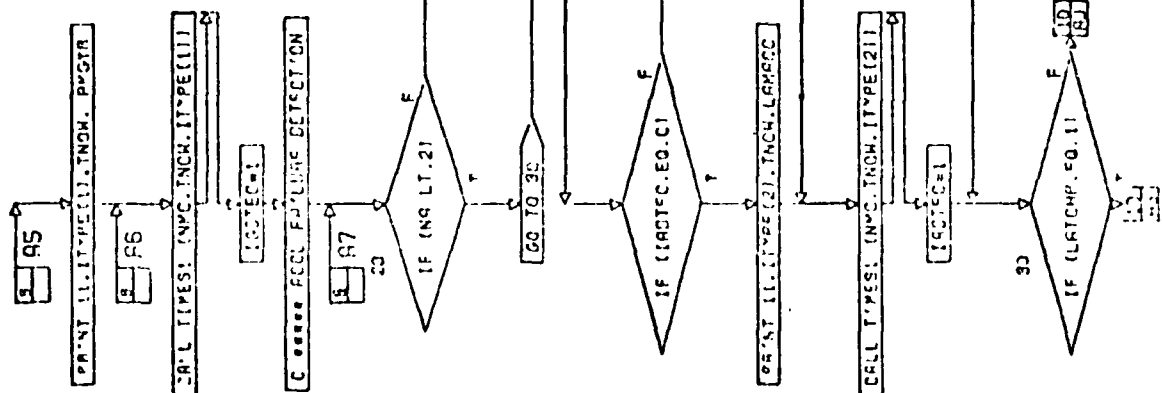


SPAY 5 07 15

ORIGINAL PAGE IS  
OF POOR QUALITY

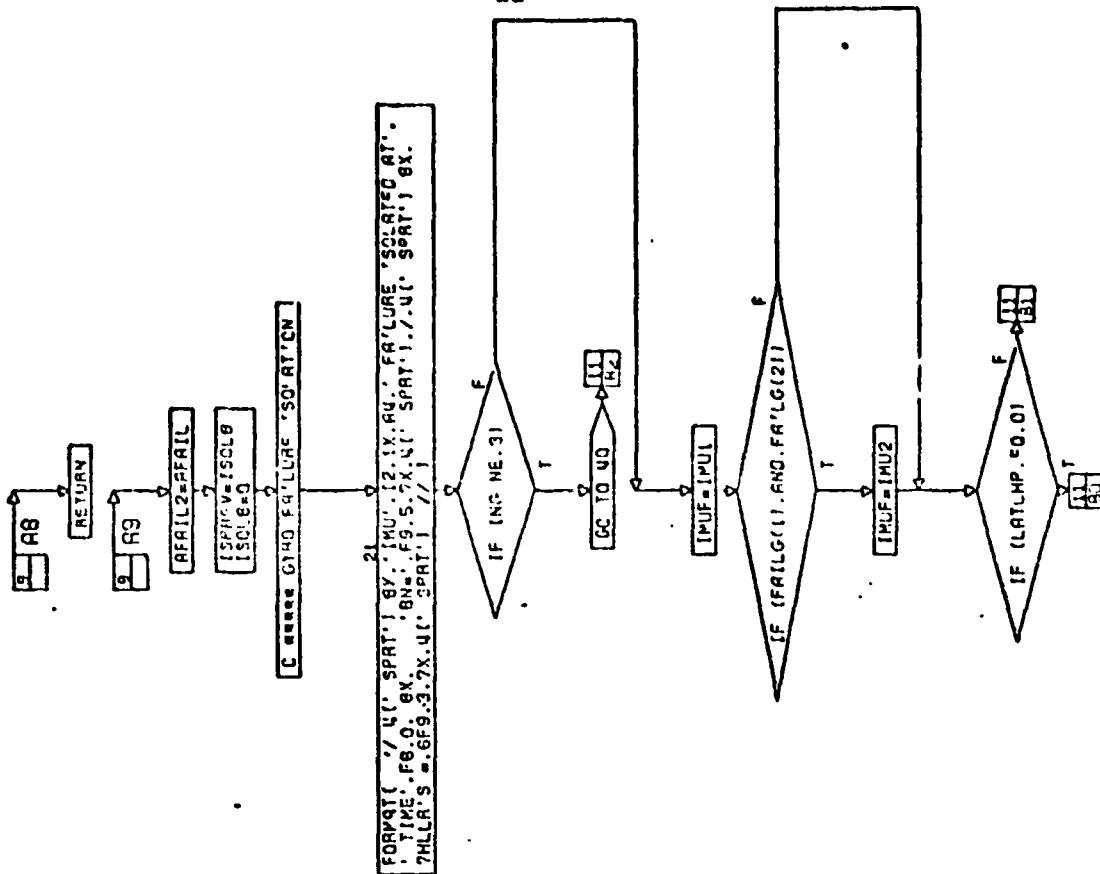


ORIGINAL PAGE IS  
OF POOR QUALITY



SPAT  
PG. 9 OF 15

CONT. ON PG 10

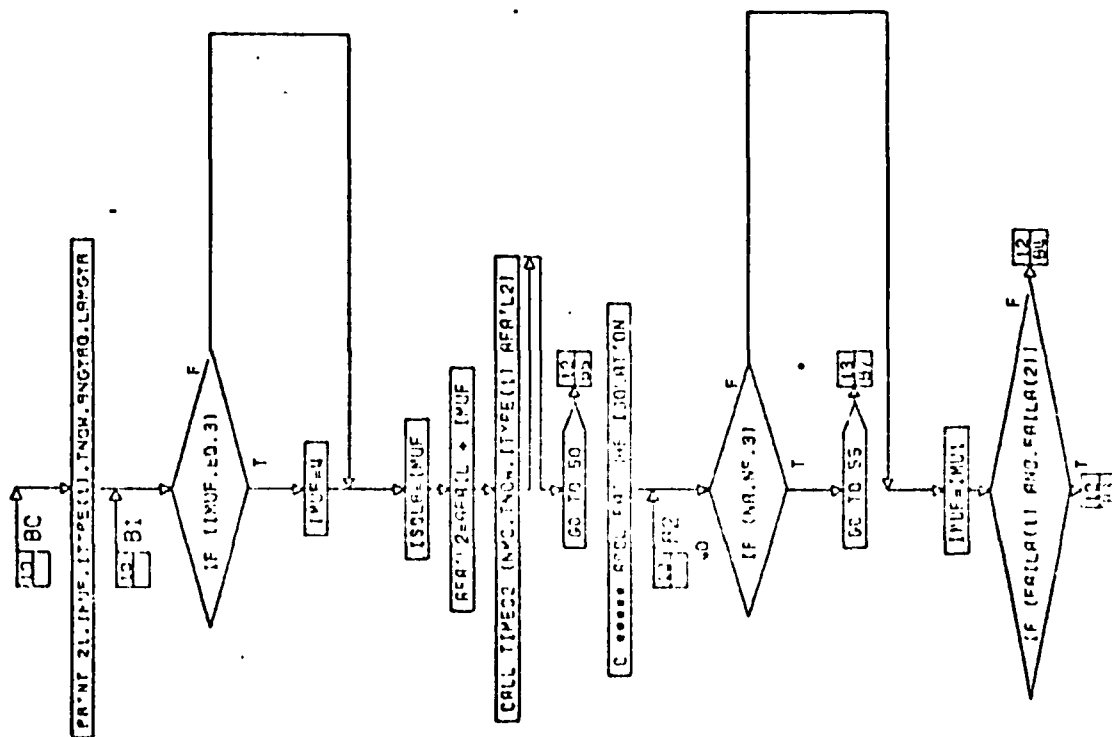


SPAT  
PG. 10 OF 15

CONT. ON PG 11

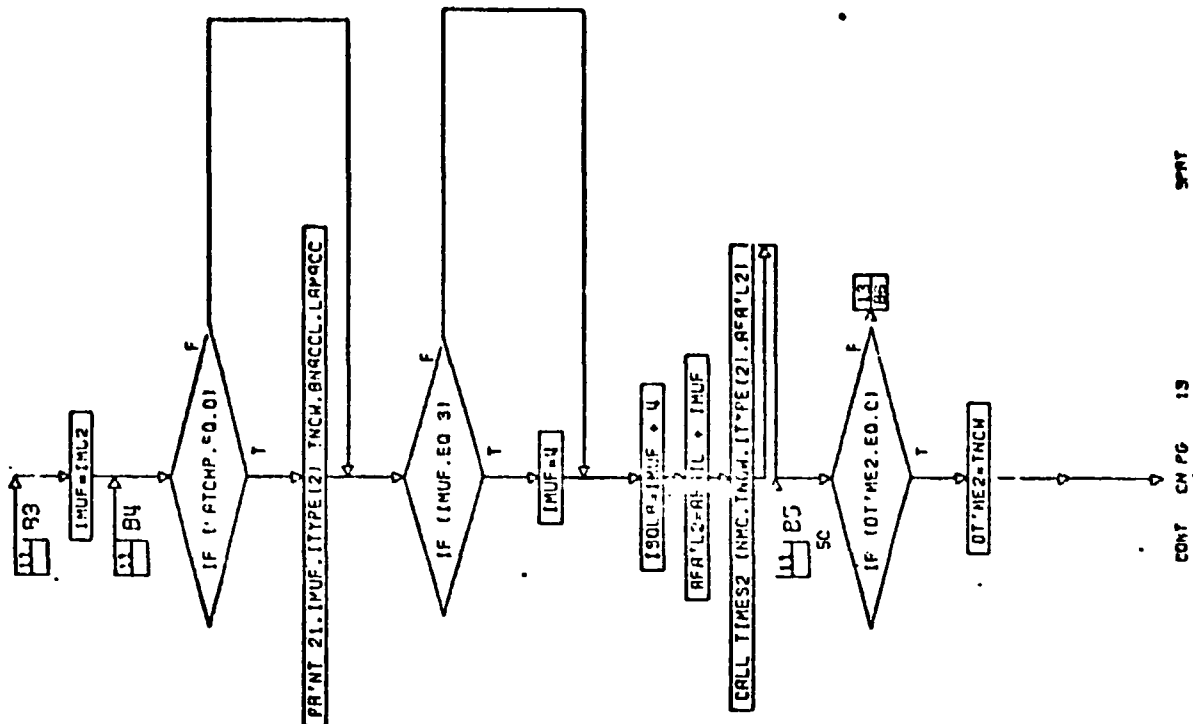


ORIGINAL PAGE IS  
OF POOR QUALITY



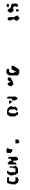
CONT. ON PG 12

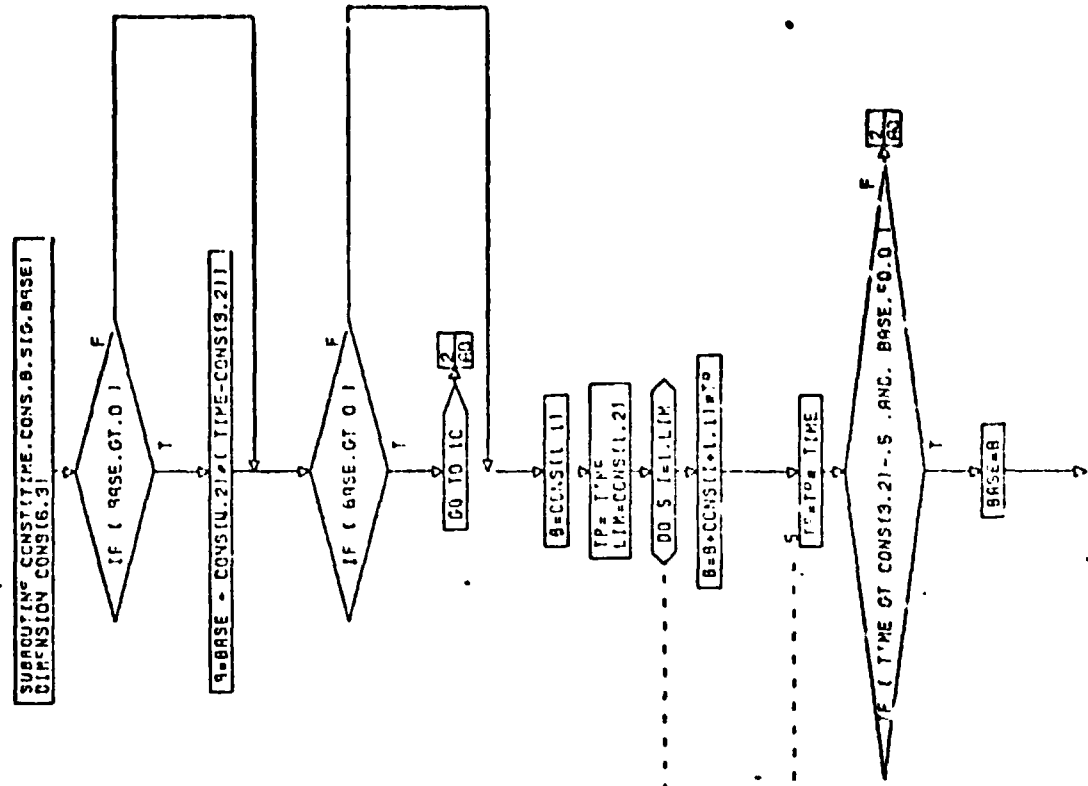
SPAT PG 12 OF 15



CONT. ON PG 13

SPAT PG 13 OF 15







# SUBROUTINES CALLED BY SPRT

## 1) DMTXH ( $M_1, M_2, M_3$ )

Matrix transposed times a matrix.

$$M_3 = M_1^T M_2$$

## 2) DMXHT ( $M_1, M_2, M_3$ )

Matrix times matrix transposed

$$M_3 = M_1 M_2^T$$

## 3) DMTXV ( $M_1, V_2, V_3$ )

Matrix transposed times a vector

$$V_3 = M_1^T V_2$$

## 4) DMXV ( $M_1, V_2, V_3$ )

Matrix times a vector

$$V_3 = M_1 V_2$$

## 5) DVSUB ( $V_1, V_2, V_3$ )

Vector subtraction

$$V_3 = V_1 - V_2$$

The above subroutines assume that all vectors and matrices are dimensioned (3) and (3,3), respectively.

## 6) TIMES1 and TIMES2 are used by the driver program to provide a summary of detection and isolation times at the end of a multi Monte Carlo cycle run.

## 7) DVSCLR (SCALAR<sub>1</sub>, $V_2, V_3$ )

Scalar times a vector

$$V_3 = \text{SCALAR}_1 V_2$$

8) DVADD ( $V_1$ ,  $V_2$ ,  $V_3$ )

Vector Addition

$$V_3 = V_1 + V_2$$